Rapid Merger of Binary Primordial Black Holes

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We propose a new scenario for the evolution of a binary of primordial black holes (PBHs). We consider the dynamical friction by ambient dark matter and gravitational interaction between a binary and a circumbinary disk, assuming PBHs do not constitute the bulk of dark matter. After the turnaround, a PBH binary loses the energy and angular momentum by the two processes, which are very effective for a typical configuration. Finally the binary coalesces due to the emission of gravitational waves in a time scale much shorter than the age of the universe. We estimate the density parameter of the resultant gravitational wave background. Astrophysical implication concerning supermassive black holes is also discussed.

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Introduction. Primordial black holes (PBHs) are hypothetical objects formed due to gravitational collapse of density perturbations at the early phase of the universe [1, 2]. A PBH mass larger than $\sim 10^{-19} M_\odot$, where $M_\odot$ is the solar mass, is enough to prohibit evaporation via the Hawking radiation [3] and PBHs with such a mass would remain as massive compact halo objects (MACHOs) until now. The possibility that PBHs consitute a certain fraction of dark matter has been investigated by many authors (see [4] and references therein).

In [5], Nakamura et al. considered the formation and evolution of PBH binaries. If PBHs were formed randomly rather than uniformly in space, some pairs of PBHs would have sufficiently small separations to overcome the cosmological expansion and then form a bound system, binary. They estimated gravitational waves from the coalescence of the binaries following the binary evolution taking gravitational wave emission into account. Because gravitational radiation is not effective to extract angular momentum from a binary unless the separation is very small, only a small fraction of binaries coalesces during the age of the universe. Nontheless, cosmological gravitational wave background amounts to $\Omega_{\rm gw} \sim 10^{-10}$ at the LISA band ($\nu_{\rm gw} \sim 10^{-4}$ Hz), where $\Omega_{\rm gw}$ and $\nu_{\rm gw}$ are the density parameter and frequency of gravitational waves, respectively [6] (see also [7]).

In this *Letter*, we consider two processes which affect the evolution of a PBH binary. One is the dynamical friction by ambient dark matter and another is the interaction between a binary and a gaseous disk surrounding it (i.e. circumbinary disk). Both processes are inevitable and help the binary shrink. Our scenario predicts that PBH binaries generically coalesce during the age of the universe and gravitational waves are emitted more efficiently compared to the result in [6]. Throughout this paper, we adopt the cosmological parameters obtained by 5-yr WMAP [8].

Binary PBH. First we give a summary on the evolution scenario of PBH binary presented by [5, 9], extending to a case where PBHs do not consitute the bulk of dark matter. For simplicity, we assume a primordial spectrum of density fluctuations which has a sharp peak with a sufficient amplitude to form PBHs so that they have a monochromatic mass function (see, e.g., [10] for such an inflationary model). The radiation and matter inside a horizon gravitationally collapse into a black hole if the overdensity is large enough there. The black hole mass, $M_{\rm bh}$, is comparable to the horizon mass and can be written as $M_{\rm bh} \sim (T_{\rm bhf}/1~{\rm GeV})^{-2} M_\odot$, where $T_{\rm bhf}$ is the temperature of the universe at the formation [11]. We assume that PBHs constitute a fraction, f, of the dark matter, that is, $\Omega_{\rm bh} = f\Omega_{\rm DM}$ where $\Omega_{\rm bh}$ and $\Omega_{\rm DM}$ are the current density parameter of PBHs and dark matter, respectively. The value of f is constrained for a wide range of $M_{\rm bh}$ and can be as large as 0.1 for $M_{\rm bh} \lesssim 0.1 M_\odot$ while much severer constraints, $f \lesssim 10^{-8}$, are obtained for $M_{\rm bh} \gtrsim 10^3 M_\odot$ [4].

Noting that the density of PBHs can be written as $\rho_{\rm bh}(T) = f\rho_{\rm DM}(T) = fT_{\rm eq}T^3$ where $T_{\rm eq} \sim 1$ eV is the temperature at the matter-radiation equality, the average separation of PBHs at the formation, $\bar{r}$, is expressed as

$$\bar{r} \sim \left(\frac{M_{\rm bh}}{\rho_{\rm bh}(T_{\rm bhf})}\right)^{1/3} \sim 10^{-11} f^{-1/3} \left(\frac{M_{\rm bh}}{1M_\odot}\right)^{5/6} {\rm pc}. \quad (1)$$

Actually, PBHs are randomly distributed in space at the formation and we consider a pair of nearest PBHs with the initial separation parametrized as $\alpha\bar{r}$. Statistically, most of nearest PBH pairs will have $\alpha \lesssim 1$ while only a few would have $\alpha \ll 1$. In fact, the probability density distribution function is $dP/d\alpha \sim 3\alpha^2 e^{-\alpha^3}$ for a random distribution as given by [5].

After the black hole formation, the separation of the pair evolves in proportion to the scale factor due to the

cosmic expansion. However, if the local energy density of the pair, $\rho_{\rm pair}=\alpha^{-3}\rho_{\rm bh}$, becomes larger than the radiation energy density, ρ_γ, the pair decouples from the cosmic expansion and forms a gravitationally bound system. We see that the turnaround occurs at the temperature,

$$T_{\rm ta}=\left(\frac{\alpha^3}{f}\right)^{-1}T_{\rm eq}. \tag{2}$$

Here it should be noted that for $\alpha>f^{1/3}$ we have always $\rho_{\rm pair}<\rho_{\rm DM}$ so that the turnaround never occurs. Thus we obtain a condition for α that a pair forms a bound system, $\alpha_0\leq\alpha\leq f^{1/3}$, where $\alpha_0=f^{1/3}M_{\rm Pl}^{-1/2}T_{\rm eq}^{1/3}M_{\rm bh}^{1/6}\sim 2\times10^{-3}f^{1/3}(M_{\rm bh}/1M_\odot)^{1/6}$ and the lower bound comes from the requirement that $\alpha\bar{r}$ should be greater than the Schwarzschild radius of a PBH. The separation at the turnaround is,

$$\begin{aligned} r_{\rm ta} &= \frac{T_{\rm bhf}}{T_{\rm ta}}\alpha\bar{r}=\alpha^4 f^{-4/3}T_{\rm eq}^{-4/3}M_{\rm bh}^{1/3} \\ &\sim 10^{-2}\left(\frac{\alpha^3}{f}\right)^{4/3}\left(\frac{M_{\rm bh}}{1M_\odot}\right)^{1/3}{\rm pc}. \end{aligned} \tag{3}$$

After the turnaround, if the two black holes had no relative velocity, they would coalesce to form a single black hole. However, as discussed in [5, 9], the tidal force from neighboring black holes would give the pair some angular momentum and prevent the coalescence. From the above arguments, the typical separation between binaries is given by $\alpha\sim f^{1/3}$ while another PBH which gives angular momentum to the binary would be typically separated from the binary by $\sim\bar{r}$. The semi-major axis of the binary is roughly given by $a\sim r_{\rm ta}$. On the other hand, the acceleration due to the tidal force and free fall time are estimated as,

$$a_{\rm tidal}\sim\frac{GM_{\rm bh}}{R^2}\frac{r_{\rm ta}}{R},\quad t_{\rm ff}\sim\sqrt{\frac{r_{\rm ta}^3}{GM_{\rm bh}}}, \tag{4}$$

respectively, where $R\equiv(T_{\rm bhf}/T_{\rm ta})\bar{r}$ is the typical separation between the binary and the third PBH at the turnaround of the pair. Thus the semi-minor axis is estimated as $b\sim a_{\rm tidal}t_{\rm ff}^2\sim r_{\rm ta}^4/R^3$. We can see that the typical eccentricity of the binary is $e=\sqrt{1-b^2/a^2}\sim\sqrt{1-f^2}$.

Once a binary is formed, it emits gravitational waves and the separation shrinks gradually. The coalescence timescale of a binary PBH is given by [12],

$$\begin{aligned} t_{\rm gw} &= \frac{5a^4}{64G^3M_{\rm bh}^3}(1-e^2)^{7/2} \\ &\sim 10^{32}f^7\left(\frac{\alpha^3}{f}\right)^{37/3}\left(\frac{M_{\rm bh}}{1M_\odot}\right)^{-5/3}\left(\frac{a}{r_{\rm ta}}\right)^4{\rm yr}. \end{aligned} \tag{5}$$

Thus we see that the emission of gravitational waves is not effective for most of the binaries ($\alpha\lesssim f^{1/3}$) to merge during the age of the universe, unless f is very small $f\lesssim10^{-3}(M_{\rm bh}/1M_\odot)^{5/21}$. However, as we see below, considering two other processes which extract the energy and angular momentum from the binary, its evolution drastically changes. Hereafter we consider a typical pair with $\alpha\lesssim f^{1/3}$. Note that if f is very small, the number of PBH binaries is very small so that the total amount of gravitational waves is also small even if most of PBH binaries merge during the age of the universe.

Interaction with dark matter. In the case of $f\ll1$ we consider here, dark matter other than PBHs is abundant and will accrete onto PBHs forming halos around them. The separation of a binary in a halo decays due to the dynamical friction from dark matter. Below we argue how effectively this happens.

At the turnaround of a pair of PBHs, each PBH would have a halo with the density $\sim\rho_{\rm DM}(z_{\rm ta})$ and mass, $M_{\rm halo}\sim\rho_{\rm DM}r_{\rm ta}^3\sim(\alpha^3/f)M_{\rm bh}$, which is comparable to the PBH mass. It would be reasonable to assume that, at the turnaround, two halos merge into a single halo which surrounds the binary. Further, ambient dark matter continues to accrete onto the binary. In matter dominant era, the halo mass and turnaround radius of dark matter evolve as $M_{\rm halo}\sim M_{\rm bh}(1+z_{\rm eq})/(1+z)$ and $r_{\rm ta,dm}\sim10^{-2}(M_{\rm bh}/1M_\odot)^{1/3}((1+z)/(1+z_{\rm eq}))^{-4/3}$ pc, respectively [13], where $z_{\rm eq}\sim3000$ is the redshift at the matter-radiation equality.

A binary in a halo shrinks by giving angular momentum and energy to dark matter particles. However, the binary-dark matter interaction can happen only if dark matter particles pass enough close to the binary, in other words, if they enter the *loss cone* [14]. Also, how much the binary shrinks depends on the dark matter mass which enter the loss cone. Here we simply assume that a dark matter particle enters the loss cone if its pericenter is smaller than the semi-major axis of the binary. Let us evaluate the orbital elements of a typical dark matter particle. As in the case of a PBH pair, dark matter receives tidal force from nearby PBHs. The semi-major axis and semi-minor axis are again estimated as $\sim r_{\rm ta,dm}$ and $\sim r_{\rm ta,dm}^4/R^3$, respectively. Thus, the pericenter can be evaluated as,

$$d\sim\frac{r_{\rm ta,dm}^7}{R^6}\sim10^{-2}f^2\left(\frac{1+z}{1+z_{\rm eq}}\right)^{-10/3}\left(\frac{M_{\rm bh}}{1M_\odot}\right)^{1/3}{\rm pc}. \tag{6}$$

From $d<r_{\rm ta}$, we obtain the following condition:

$$\frac{1+z}{1+z_{\rm eq}}>f^{3/5}\left(\frac{\alpha^3}{f}\right)^{-2/5}. \tag{7}$$

This means that dark matter which turnarounds before this redshift enters the loss cone. The total mass which enters the loss cone is then bounded by,

$$M_{\rm lc}<f^{-3/5}\left(\frac{\alpha^3}{f}\right)^{2/5}M_{\rm bh}. \tag{8}$$

Next we consider the time scale of the dynamical friction. It depends on the dark matter density of the halo and we assume it to be equal to the average density of the universe. In fact, the dark matter would be denser in a halo and this assumption overestimates the timescale, which leads to a conservative result. The time scale is given by [15],

$$t_{\rm df} \sim \frac{v_{\rm bh}^3}{4\pi G^2 M_{\rm bh}\rho_{\rm DM}\ln\Lambda}\left[{\rm erf}(X)-\frac{2X}{\sqrt{\pi}}e^{-X^2}\right]^{-1} \quad (9)$$

where erf is the error function, $\ln\Lambda \approx 10$ is the Coulomb logarithm. Here $X \equiv v_{\rm bh}/(\sqrt{2}\sigma)$ where $v_{\rm bh}$ and σ are the black hole velocity and velocity dispersion of dark matter, respectively. We conservatively assume that the dynamical friction is effective only when this time scale is smaller than the Hubble time at that time. In matter dominated era, the ratio is,

$$Ht_{\rm df} \sim \frac{1}{\Gamma}\left(\frac{1+z}{1+z_{\rm eq}}\right)^{-3/2}, \quad (10)$$

where $\Gamma \sim 20$ is a numerical factor and we set $v_{\rm bh}$ and σ to the Keplerian velocity $\sqrt{GM_{\rm bh}/r_{\rm ta}}$. Thus the dynamical friction is effective for $1+z > \Gamma^{-2/3}(1+z_{\rm eq})$. The total mass which has entered the loss cone by this time is estimated as $M_{\rm lc} \sim \Gamma^{2/3}M_{\rm bh}$. Together with the argument around Eq. (8), our estimation of the total mass which contributes to the dynamical friction is $M_{\rm lc} \sim \min[f^{-3/5}M_{\rm bh}, \Gamma^{2/3}M_{\rm bh}]$, which is $\Gamma^{2/3}M_{\rm bh} \sim 7M_{\rm bh}$ for $f \lesssim 0.04$.

Once we obtain the loss cone mass, we can evaluate how much the binary shrinks, using the fact that the mean energy which dark matter of mass $dM_{\rm lc}$ extracts from a binary is $\sim GM_{\rm bh}dM_{\rm lc}/a$:

$$\frac{GM_{\rm bh}}{a}dM_{\rm lc} \approx GM_{\rm bh}^2 d\left(\frac{1}{a}\right). \quad (11)$$

Thus, the resultant separation after the dynamical friction ceases, $a_{\rm df}$, is given by,

$$\frac{r_{\rm ta}}{a_{\rm df}} \approx \exp\left(\frac{M_{\rm lc}}{M_{\rm bh}}\right). \quad (12)$$

Adopting a typical value given above, $M_{\rm lc} \sim 7M_{\rm bh}$, we see that the binary shrinks by about three orders by $z \sim 400$. Here it should be noted that, in the above analysis, we used several simplifying assumptions. In particular, there would be a large uncertainty in the estimation of the shrinking factor. Nevertheless, we believe that the dynamical friction would have an important effect on the evolution of PBH binaries unless f is close to unity. More robust evaluation of the shrinking factor would need N-body simulations, which we will present elsewhere.

Interaction with gas. Baryon gas, as well as the dark matter, also accretes onto the PBHs. Even if the gas has negligible angular momentum with respect to the center of mass of the binary, the gas can not fall directly into the black holes but instead would form a rotating disk around the binary (i.e. a circumbinary disk). The circumbinary disk can play a role of an efficient mechanism to extract the angular momentum from a binary [16, 17, 18, 19]. This is because the circumbinary disk and binary exchange their masses and angular momenta through the tidal-resonant interaction [19, 20, 21, 22].

Neglecting the angular momentum of the gas, we consider the Bondi accretion and its accretion radius is given by

$$r_{\rm B} = \frac{GM_{\rm bh}}{c_\infty} \sim 4\times10^{-5}\frac{M_{\rm bh}}{1M_\odot}\frac{1+z_{\rm eq}}{1+z}\ {\rm pc}, \quad (13)$$

where $c_\infty \sim 10(1+z)/(1+z_{\rm eq})$ km/s is the average sound velocity [23]. When the Bondi radius is smaller than the separation of the binary, the gas accretes onto each PBH separately and a circumbinary disk would not form. Because the Bondi radius increases in time while the binary separation would shrink due to the dynamical friction, the Bondi radius exceeds the separation sooner or later and a circumbinary disk would form. If f is relatively small $f \lesssim 0.1$ and the dynamical friction is as effective as we argued in the previous section, a circumbinary disk would begin to form at $z \lesssim 1000$.

According to [4, 23], the gas accretion rate onto a black hole with $1M_\odot$ has a peak around the matter-radiation equality time and the peak accretion rate is several percent of the Eddington rate: $\dot{M}_{\rm Edd} \sim 2\times10^{-9}(M_{\rm bh}/1M_\odot)M_\odot$/yr. For $M_{\rm bh} < 1M_\odot$, the mass of the circumbinary disk $M_{\rm cbd} \sim \dot{M}_{\rm Edd}H^{-1}$ can be well fitted by $M_{\rm cbd}/M_{\rm bh} \sim 10^{-5}(M_{\rm bh}/1M_\odot)$. Considering only the interaction between the binary and the circumbinary disk, that is, neglecting the effects of the respective disks, the time scale of the orbital decay can be estimated as [19],

$$\begin{aligned} t_{\rm cbd} &= t_{\rm vis}\frac{M_{\rm bh}}{M_{\rm cbd}} \\ &\sim 7\times10^6\left(\frac{a}{10^{-4}\ {\rm pc}}\right)^{1/2}\left(\frac{M_{\rm bh}}{1M_\odot}\right)^{-1}{\rm yr}, \end{aligned} \quad (14)$$

where $t_{\rm vis} \sim 7\times10^3(a/1{\rm pc})^{1/2}$yr is the viscous time scale of the circumbinary disk with an assumption that the temperature of the inner edge of the circumbinary disk is $(1+z)T_0$ and the Shakura-Sunyaev viscosity parameter is 0.1 [24]. Further we used a typical parameter set, $a_{\rm df} \sim 10^{-4}$ pc and $1+z \sim 400$ taking the dynamical friction and time evolution of the Bondi radius into account. We will use this parameter set below as well.

As the separation decreases, the timescale (14) also decreases gradually while the timescale of gravitational waves, Eq. (5), decreases much more rapidly. When the two time scales become comparable, the most effective process of extracting angular momentum from the binary

switches to the gravitational wave emission. The critical separation, a_c, is,

$$a_c \sim 10^{-9} \left(\frac{M_{bh}}{1M_\odot}\right)^{4/21} \text{pc}, \tag{15}$$

and the critical redshift, z_c, which is determined by $H^{-1} = t_{cbd}$ is given by,

$$1 + z_c \sim 100 \left(\frac{M_{bh}}{1M_\odot}\right)^{2/3}. \tag{16}$$

Here we assumed the binary orbit is circular after the dynamical friction. Substituting Eq. (15) into Eq. (5), the time scale of gravitational radiation reduces to $t_{gw} \sim 10^4 (M_{bh}/1M_\odot)^{-19/21}$yr. Thus, for the typical parameter set, PBH binaries of $M_{bh} \gtrsim 0.01 M_\odot$ generically coalesce by around $z_c > 5$. This is to be contrasted with [5] where only a small fraction of binary PBHs coalesces during the age of the universe.

Summary and Discussions. We have studied the evolution of PBH binaries by considering the gravitational interactions with dark matter and baryon gas. Let us summarize our scenario in turn. After the turnaround of a pair of PBHs ($z \gtrsim z_{eq}$), dark matter accretes onto the pair to form a halo. In the early phase ($z \lesssim z_{eq}$), the dark matter density is sufficiently high that the dynamical friction is effective. Although a precise estimation of the shrinking factor of the binary separation is hard analytically, our simple model implies that a binary can shrink by a couple of orders. Some time after the recombination ($z \lesssim 1000$), a circumbinary disk would form and extract angular momentum from the binary. At $z \sim O(100)$, the most effective process extracting angular momentum from the binary switches to the gravitational wave emission. Finally the binary would coalesce at around $z \sim 100$.

If our scenario works, most of PBH binaries coalesce around $z = z_c$ emitting gravitational waves. Thus we can expect a substantial amount of the gravitational wave background. Below we present an order-of-magnitude estimation simply assuming that all binaries coalesce at $z = z_c$. Given the probability distribution function for the initial separation, $dP/d\alpha \sim 3\alpha^2 e^{-\alpha^3}$, we see that the fraction of PBHs which form a binary is $P(\alpha < f^{1/3}) \sim f$ assuming $f \ll 1$. Then, at $z = z_c$, the number of binaries in a horizon volume is given by $\sim f^2 H^{-3} \rho_{DM}/M_{bh}$. Noting that the energy of gravitational waves emitted by a binary is $\sim (GM_{bh}\nu)^{2/3} M_{bh}$, where $\nu = \sqrt{GM_{bh}/a^3}$ is the frequency of the gravitational wave determined by the binary separation, the energy density of the gravitational waves at $z = z_c$ is $\epsilon_{gw}(\nu, z_c) \sim (GM_{bh}\nu)^{2/3} f^2 \rho_{DM}(z_c)$. Finally, the current density parameter of the gravitational waves is given by,

$$\begin{aligned}\Omega_{gw,0}(\nu) &= \frac{\epsilon_{gw}((1+z_c)\nu, z_c)}{(1+z_c)^4 \rho_c} \\ &\sim 4 \times 10^{-7} f^2 \left(\frac{\nu}{\nu_{c,0}}\right)^{2/3} \left(\frac{M_{bh}}{M_\odot}\right)^{37/63}, \end{aligned} \tag{17}$$

where $\nu_{c,0} \sim 5 \times 10^{-2} (M_{bh}/M_\odot)^{3/14} (a/a_c)^{-3/2}/(1 + z_c)$Hz represents the lowest frequency corresponding to the separation a_c. The frequency domain is given as $\nu_{c,0} < \nu < \nu_{mso,0}$ where $\nu_{mso,0} \sim 10^5 (M_{bh}/M_\odot)^{-1}/(1 + z_c)$Hz, which is the frequency corresponding to the marginally stable orbit of a PBH binary. The gravitational wave background with $f < 0.03$ is estimated as $\Omega_{gw,0} \sim 10^{-10}$, which is comparable with that estimation by [5] corresponding to $f = 1$, and is enough large to be detected with LISA, DECIGO and LIGO. Thus the observation of the gravitational wave background would give a strong constraint on the dark matter fraction of PBHs with $M_{bh} \lesssim 1M_\odot$.

Finally let us mention another astrophysical implication of our scenario. If the spectrum of density perturbation is broad rather than monochromatic as assumed here, clusters of PBHs can be formed [25]. If this is the case, PBH mergers may occur frequently in clusters to form much larger black holes than the original PBHs. This may help explain the presence of a supermassive black hole recently discovered at $z \sim 6$ [26, 27], providing supermassive black holes directly by successive merger of PBHs or smaller "seed" black holes.

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